

Chiral perturbation theory ^{*} [†]J. Gasser^a

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Abstract

I present an outline of chiral perturbation theory and discuss some recent developments in the field.

Pacs: 11.30.Rd, 12.39.Fe

Keywords: Chiral Symmetry, Chiral Perturbation Theory

1. EFFECTIVE THEORY

The QCD lagrangian can be replaced at low energy with an effective lagrangian that is formulated in terms of the asymptotically observable fields [1–3]. This effective lagrangian reads for processes with pions alone

$$\mathcal{L}_M = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + M^2 (U + U^\dagger) \rangle. \quad (1)$$

Here, the matrix field U is an element of $SU(2)$, and the symbol $\langle A \rangle$ denotes the trace of the matrix A . In the following, I use the parameterization

$$U = \sigma + \frac{i\phi}{F}; \quad \phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad (2)$$

$$\sigma = [1 - \phi^2/F^2]^{\frac{1}{2}}, \quad (2)$$

^{*}Invited talk given at the QCD Euroconference 99, Montpellier, 7-13th July, 1999. The present article contains additional references as compared to the version that will appear in the proceedings.

[†]This work was supported in part by the Swiss National Science Foundation, and by TMR, BBW-Contract No. 97.0131 and EC-Contract No. ERBFMRX-CT980169 (EURODAΦNE).

and the notation

$$\phi = \sum_{i=1}^3 \tau^i \phi_i, \quad \boldsymbol{\pi} = (\phi_1, \phi_2, \phi_3). \quad (3)$$

The coupling constant $F \simeq 93$ MeV measures the strength of the $\pi\pi$ interaction, and the quantity M^2 denotes the square of the physical pion mass (that I denote with M_π) at lowest order in an expansion in powers of $1/F$, see below. It is proportional to the light quark masses m_u, m_d ,

$$M^2 = 2\hat{m}B, \quad \hat{m} = \frac{1}{2}(m_u + m_d), \quad (4)$$

where B itself is related to the quark condensate, see [2]. Note that the quantity M^2 occurs not only in the kinetic term of the pion lagrangian, but also in the interaction: it acts both as a mass parameter and as a coupling constant. The lagrangian \mathcal{L}_M is called the "non-linear sigma-model lagrangian". This name has led to some confusion in the literature about the meaning of the effective lagrangian: one is not replacing QCD with a "chiral model", as this procedure is often called. To the contrary, \mathcal{L}_M can be used to calculate processes at low energies, with a result that is identical to the one in QCD [1–3].

In case we wish to consider also nucleons, one has to enlarge the above lagrangian. Let us consider processes where a single baryon (proton or neutron) travels in space, emitting and absorbing pions in all possible ways allowed by chiral symmetry. This process is illustrated in Fig. 1. One need not consider processes with closed nu-

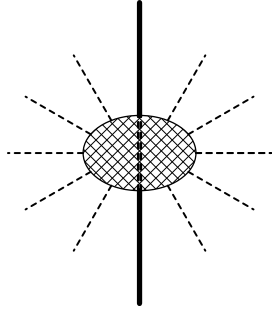


Figure 1. The nucleon traveling through space, emitting and absorbing pions.

cleon lines. These contributions may be absorbed in a renormalization of the coupling constants in the effective lagrangian for meson-nucleon interactions,

$$\mathcal{L}_{MB} = \bar{\Psi} \left\{ i \not{D} - m + \frac{1}{2} g_A \not{\gamma}_5 \right\} \Psi, \quad (5)$$

with

$$u^2 = U, u_\mu = i u^\dagger \partial_\mu U u^\dagger, \quad \Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u], D_\mu = \partial_\mu + \Gamma_\mu. \quad (6)$$

Here, Ψ denotes the nucleon field, m is the nucleon mass in the chiral limit $m_u = m_d = 0$, and g_A is the neutron decay constant $g_A \simeq 1.25$. The effective lagrangians (1),(5) contain the three couplings $1/F, M^2, g_A$ and the nucleon mass m as free parameters. These couplings are not fixed by chiral symmetry,

$$F = c_1 \Lambda_{QCD}, m = c_2 \Lambda_{QCD} \dots, \quad (7)$$

where Λ_{QCD} is the renormalization group invariant scale of QCD, and where c_i are dimensionless numbers that can in principle be calculated in QCD. On the other hand, there are *relations* between *physical* quantities, e.g., the famous Goldberger Treiman relation

$$g_{\pi N} = \frac{m g_A}{F}. \quad (8)$$

The quantities $g_{\pi N}, F, m$ and g_A are evaluated in the chiral limit $m_u = m_d = 0$. In the real world, there are corrections of order m_u, m_d to this relation [4].

2. TREE GRAPHS

Tree graphs evaluated with (1),(5) generate the leading order term in a systematic low-energy expansion of the S -matrix elements [1-3]. I illustrate this fact with two examples.

2.1. The pion mass

It suffices to consider the terms in \mathcal{L}_M that are quadratic in the pion fields,

$$\mathcal{L}_M = \frac{1}{2} [\partial_\mu \pi \cdot \partial^\mu \pi - M^2 \pi^2] + \dots. \quad (9)$$

Therefore, the effective theory contains at tree level three mass degenerate bosons π^+, π^-, π^0 , with

$$M_{\pi^+}^2 = M_{\pi^-}^2 = M_{\pi^0}^2 = M^2. \quad (10)$$

At the leading order considered here, there is no isospin splitting: the masses of the charged and of the neutral pion are identical, see [1]. A small mass difference due to $m_u \neq m_d$ does show up only at next order in the chiral expansion.

2.2. $\pi\pi$ scattering

The full power of the effective lagrangian method comes into play when one starts to evaluate scattering matrix elements. Consider for this purpose elastic $\pi\pi$ scattering. The interaction part of the effective lagrangian is

$$\mathcal{L}_{int} = \frac{1}{8F^2} \{ \partial_\mu \pi^2 \partial^\mu \pi^2 - M^2 (\pi \cdot \pi)^2 \} + \dots. \quad (11)$$

Since we calculate tree matrix elements, the terms at order $O(\pi^6)$ - indicated by the ellipses - are not

needed. The contributions with four fields in the lagrangian contain two types of vertices: the first one has two derivatives, while the second contains the parameter M^2 as a coupling constant. In the following I consider the isospin symmetry limit $m_u = m_d$ and use the standard notation

$$T^{ab;cd} = \delta^{ab;cd} A(s, t, u) + \text{cycl.} \quad (12)$$

for the matrix element of the process

$$\pi^a(p_1)\pi^b(p_2) \rightarrow \pi^c(p_3)\pi^d(p_4), \quad (13)$$

with the Mandelstam variables

$$\begin{aligned} s &= (p_1 + p_2)^2, t = (p_1 - p_3)^2, \\ u &= (p_1 - p_4)^2. \end{aligned} \quad (14)$$

The result of the calculation is

$$A \stackrel{\text{tree}}{=} \frac{s - M^2}{F^2} \stackrel{\text{tree}}{=} \frac{s - M_\pi^2}{F_\pi^2}. \quad (15)$$

The second equal sign in Eq. (15) is based on the fact that the coupling M^2 can be replaced at tree level with the square of the physical pion mass, see Eq. (10), and that the physical pion decay constant F_π is equal to F in the same approximation. Of course, the result Eq. (15) agrees with the expression evaluated [5] with current algebra techniques a long time ago.

In order to compare the above expression for the scattering matrix element with the data, it is useful to consider the partial wave expansion of the amplitude. I consider the isospin zero combination

$$\begin{aligned} T^0(s, t) &= 3A(s, t, u) + A(t, u, s) + A(u, s, t), \\ s &= 4(M_\pi^2 + p^2), t = -2p^2(1 - \cos \theta), \end{aligned} \quad (16)$$

where θ is the scattering angle in the center of mass system, and p^2 is the square of the pion momentum. $T^0(s, t)$ may be expanded in Legendre polynomials,

$$T^0(s, \theta) = 32\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) t_l^0(s), \quad (17)$$

with energy-dependent coefficients $t_l^0(s)$. Unitarity implies that, in the elastic region $4M_\pi^2 \leq s \leq 16M_\pi^2$, the coefficients have the structure

$$\begin{aligned} t_l^0(s) &\stackrel{\text{unitarity}}{=} \frac{1}{\sigma} e^{i\delta_l^0(s)} \sin \delta_l^0(s), \\ \sigma &= (1 - 4M_\pi^2/s)^{1/2}, \end{aligned} \quad (18)$$

with real phase shifts δ_l^0 . Therefore, knowing $A(s, t, u)$, one may evaluate T^0 , then t_l^0 and finally the phase shift $\delta_l^0(s)$ in the low-energy expansion. The behavior of the partial wave amplitudes near threshold is of the form

$$\Re t_l^I(s) = p^{2l} \{a_l^I + p^2 b_l^I + O(p^4)\}. \quad (19)$$

The tree-level result (15) gives [5]

$$a_0^0 \stackrel{\text{tree}}{=} \frac{7M_\pi^2}{32\pi F_\pi^2} = 0.16, \quad (20)$$

to be compared with the observed value [6,7]

$$a_0^0 \stackrel{\text{exp.}}{=} 0.26 \pm 0.05. \quad (21)$$

3. LOOPS

The isospin zero amplitude in elastic $\pi\pi$ scattering is real at tree level,

$$T^0 \stackrel{\text{tree}}{=} \frac{2s - M_\pi^2}{F_\pi^2}. \quad (22)$$

On the other hand, unitarity (18) requires the Legendre coefficients in the partial wave expansion (17) to be complex. This apparent inconsistency arises for the following reason. CHPT represents the amplitude T^0 through an energy expansion - the tree-level result (22) is the leading order term, quadratic in the momenta. The partial waves t_l^0 are therefore also of order p^2 according to Eq. (17). One concludes furthermore from

$$\Re t_l^0 = \frac{1}{2\sigma} \sin 2\delta_l^0 \quad (23)$$

that the phase shifts δ_l^0 are of order p^2 as well. The imaginary part of the partial waves,

$$\Im t_l^0 = \frac{1}{\sigma(s)} \sin^2 \delta_l^0, \quad (24)$$

is then of order p^4 , as is the imaginary part of the amplitude T^0 . Since we have not yet considered the amplitude to this accuracy, we have simply missed its absorptive part so far.

The remedy is simple: one needs to consider loops, generated by the effective lagrangian (1) [1]. The amplitudes so evaluated satisfy unitarity in a perturbative sense. Moreover, they have the

correct analytic and crossing symmetry properties.

The expansion according to the number of independent loops of connected Feynman diagrams may be identified with an expansion in inverse powers of F^2 . Indeed, by definition, the number of independent loops generated by \mathcal{L}_M is the number of independent four-momenta in the diagrams. The effective lagrangian (1) has an infinite number of vertices. In the following I denote by V_n the number of vertices with n fields in the diagram under consideration, and by I the number of internal lines. Using energy-momentum conservation at each vertex, the number L of independent loops is

$$L = I + 1 - \sum_n V_n. \quad (25)$$

Furthermore, for connected *tree* graphs the number E of external lines is

$$E = \sum_n n V_n - 2I. \quad (26)$$

This formula is valid also for connected diagrams with loops. Finally, eliminating I in Eq. (26) with the help of Eq. (25) gives

$$E + 2L = 2 + \sum_n (n - 2) V_n. \quad (27)$$

Next, I count powers of F^2 . Each vertex that contains n fields generates a factor F^{2-n} . As a result, the overall power of F in a fixed diagram is $\sum_n (2-n) V_n$. According to Eq. (27), the diagram is therefore proportional to

$$\frac{F^{2-E}}{F^{2L}}. \quad (28)$$

This shows that the loop expansion coincides with the expansion in inverse powers of F^2 . For a connected n -point function G_n , the expansion reads

$$F^{n-2} G_n = G_{n,\text{tree}} + \frac{G_{n,1 \text{ loop}}}{F^2} + \frac{G_{n,2 \text{ loops}}}{F^4} + \dots \quad (29)$$

For dimensional reasons, n loop contributions are therefore suppressed by $2n$ powers of energy with respect to the tree diagram. In dimensional regularization, the only dimensionful parameters in

the effective theory - besides F - are the external momenta and M^2 . We conclude that the loop expansion amounts to an expansion in powers of the external momenta and of M^2 , where each term in this expansion is multiplied with a dimensionless function of the momenta and of M^2 . Therefore, the loop expansion is equivalent to an energy expansion[1].

4. EXTERNAL FIELDS

For the evaluation of loops, it is useful to introduce the concept of external fields. Let us consider QCD in the two flavor case, and define

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{QCD}^0 + \Delta\mathcal{L}, \\ \Delta\mathcal{L} &= \bar{q}\gamma_\mu [v^\mu(x) + \gamma_5 a^\mu(x)] q \\ &\quad - \bar{q}[s(x) - i\gamma_5 p(x)] q. \end{aligned} \quad (30)$$

The symbol \mathcal{L}_{QCD}^0 denotes the QCD lagrangian without the quark mass matrix. The external fields v_μ, a_μ, s and p are hermitian, color neutral two by two matrices in flavor space. In order to avoid the discussion of anomalies [8], I consider in the following only the case where the external vector and axial fields are traceless,

$$\langle v_\mu \rangle = \langle a_\mu \rangle = 0. \quad (31)$$

The generating functional Γ is given by

$$e^{i\Gamma(v,a,s,p)} = \langle 0 | T e^{i \int d^4x \Delta\mathcal{L}} | 0 \rangle. \quad (32)$$

It contains all the information on the Green functions built from vector, axial, scalar and pseudoscalar quark currents. As an example, the term linear in the scalar field,

$$\Gamma = - \int d^4x s^{\alpha\beta} \langle 0 | \bar{q}^\alpha(x) q^\beta(x) | 0 \rangle + \dots, \quad (33)$$

contains the vacuum expectation value of the quark fields, whereas the term quartic in the axial current a^μ contains the $\pi\pi$ scattering amplitude, and so on. By expanding Γ around $v^\mu = a^\mu = s = p = 0$ one generates the Green functions in the chiral limit $m_u = m_d = 0$, whereas the expansion around

$$v^\mu = a^\mu = p = 0, s = \text{diag}(m_u, m_d) \quad (34)$$

generates the Green functions at finite values of the quark masses.

The generating functional Γ contains the complete knowledge of Green function built from quark currents - it is therefore impossible to evaluate it in closed form with present techniques. On the other hand, the invariance theorem proven by Leutwyler [3] states that Γ may be evaluated at low energies using an effective theory, where only the observed asymptotic states occur in the lagrangian. In addition, the effective lagrangian may be taken to be gauge invariant by itself. The corresponding rules to evaluate the Green functions of quark currents in QCD with two flavors are discussed in the following section.

5. EFFECTIVE THEORY OF QCD

The central object in the invariance theorem [3] is the gauge invariant effective lagrangian. It consists of a series of terms, each of which is gauge invariant by itself,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots \quad (35)$$

Here, \mathcal{L}_{2n} contains m_1 derivatives and m_2 quark mass matrices, with $m_1 + 2m_2 = 2n$ (I consider here standard power counting for the chiral condensate - the generalized case is discussed below). The leading term in the low-energy expansion is obtained by evaluating tree graphs with \mathcal{L}_2 . The next-to-leading contributions are obtained by evaluating one-loop graphs with \mathcal{L}_2 and tree graphs generated by $\mathcal{L}_2 + \mathcal{L}_4$ with exactly one vertex from \mathcal{L}_4 , etc. This procedure to evaluate Green functions is called chiral perturbation theory (CHPT).

In order to construct these effective lagrangians, it is useful to first have building blocks that transform covariantly under local gauge transformations. For this purpose, one defines the transformation $h(x)$ by

$$\begin{aligned} U &\xrightarrow{G} V_R U V_L^\dagger, \\ u &\rightarrow V_R u h^\dagger, \quad u^2 = U. \end{aligned} \quad (36)$$

Furthermore, one uses the field

$$\chi = 2B(s + ip), \quad (37)$$

the covariant derivative

$$D_\mu X = \partial_\mu X - i(v_\mu + a_\mu)X + iX(v_\mu - a_\mu), \quad (38)$$

and the building blocks

$$\begin{aligned} u_\mu &= iu^\dagger D_\mu U u^\dagger = -iu D_\mu U^\dagger u = u_\mu^\dagger, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ \chi_-^\mu &= u^\dagger D^\mu \chi u^\dagger - u D^\mu \chi^\dagger u, \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u. \end{aligned} \quad (39)$$

The quantity $F_R^{\mu\nu}$ ($F_L^{\mu\nu}$) stands for the field strength associated with the nonabelian external field $v_\mu + a_\mu$ ($v_\mu - a_\mu$). Each of the above building blocks transforms as

$$I \xrightarrow{G} h I h^\dagger \quad (40)$$

under *local* gauge transformations.

The effective lagrangians $\mathcal{L}_{2,4}$ are [2]

$$\mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle, \quad (41)$$

and

$$\mathcal{L}_4 = \sum_{i=1}^7 l_i P_i + \dots, \quad (42)$$

where

$$\begin{aligned} P_1 &= \frac{1}{4} \langle u^\mu u_\mu \rangle^2, & P_2 &= \frac{1}{4} \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle, \\ P_3 &= \frac{1}{16} \langle \chi_+ \rangle^2, & P_4 &= \frac{i}{4} \langle u_\mu \chi_-^\mu \rangle, \\ P_5 &= -\frac{1}{2} \langle f_-^{\mu\nu} f_{-\mu\nu} \rangle, & P_6 &= \frac{i}{4} \langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle, \\ P_7 &= -\frac{1}{16} \langle \chi_- \rangle^2. \end{aligned} \quad (43)$$

For $a_\mu = v_\mu = p = 0, s = \text{diag}(m_u, m_d)$, the lagrangian \mathcal{L}_2 agrees with \mathcal{L}_M in (1). The ellipses in (42) denote polynomials in the external fields which are independent of the pion variables. These do not contribute to S -matrix elements. The lagrangian contains 7 low-energy constants l_i (LEC's). Some of these are divergent in four dimensions - they cancel the divergences generated by the one-loop graphs [2]. The structure of \mathcal{L}_6 will be discussed below.

6. APPLICATIONS

6.1. Theory

By use of the above technique, most matrix elements accessible to experimental data have been

evaluated to one loop accuracy, including baryons and weak interactions. It is impossible to cover here the results of these investigations. I refer the interested reader instead to the *Second DAΦNE Physics Handbook* [9] and to recent chiral workshops [10,11] for a collection of many results. In addition, let me mention the EURODAΦNE collaboration [12], where 10 European universities and research institutes have formed a network, in order to study high precision elementary particle physics at the DAFNE Φ -factory. Topics considered by the network include

- CP and CPT physics in the kaon system
- Chiral Perturbation Theory
- K and η', η decays
- K_{l3}, K_{l4} decays, $\pi\pi \rightarrow \pi\pi$
- total hadronic cross-section in electron-positron collisions below 2 GeV
- vector mesons in effective lagrangians
- nuclear physics with kaons from Φ decays

I refer the interested reader to the relevant Home Pages [12] for more information.

6.2. Experiment

There are presently several experimental activities involved in the low-energy region of the Standard Model. The following table lists some of them.

DIRAC (CERN)	$\pi^+\pi^-$ - atom $\pi\pi$ scattering lengths
KLOE (Frascati)	$K_{l3}, K_{l4}, K \rightarrow \pi\pi, \eta \rightarrow 3\pi, \dots$ $\pi\pi$ scattering lengths from K_{l4}
DEAR (Frascati)	KN - atom KN scattering lengths
MAMI (Mainz)	$\gamma N \rightarrow \pi N, \dots$
E865 (Brookhaven)	K_{l3}, K_{l4}, \dots $\pi\pi$ scattering lengths from K_{l4}
R-98-01.1 (PSI)	πp - atom πN scattering lengths

We expect that these experiments will provide us with relevant new insight into the low-energy structure of the Standard Model.

7. SOME RECENT DEVELOPMENTS

At this conference, there were several talks related to recent developments in low-energy effective theories: E. de Rafael (large N_c), L. Girlanda (chiral condensate), M. Knecht (large N_c), S. Peris (large N_c), J. Prades ($\Delta I = 1/2$ rule), H. Sazdjian (pionium), J. Soto (pionium). I refer the reader to the corresponding contributions in these proceedings for detailed information. In the following, I present several topics where progress has recently been achieved - of course, this selection is a matter of personal taste.

7.1. Effective lagrangian at $O(p^6)$

Chiral perturbation theory in the meson sector is now being carried out at next-to-next-to-leading order. Several complete two-loop calculations exist [13,14]. To relate the low-energy constants that occur at order p^6 to those appearing in other processes, one needs to know the effective lagrangian \mathcal{L}_6 in its most general form. It has been constructed recently for the general flavor case, as well as for $N_f = 2, 3$ [15,16]. In the case of N_f light flavors, there are 112 in principle measurable and 3 contact terms, that reduce to 90+4

(53+4) for 3 (2) flavors [16]. In Ref. [16], the divergence structure of \mathcal{L}_6 has been determined as well. This provides a very thorough check on any specific two-loop calculation.

The number of new couplings may seem large. On the other hand, in the chiral limit $m_u = m_d = 0$, the number of new phenomenological constants goes down [16]:

$$\begin{array}{lll}
 3 & \text{LEC's in} & \pi\pi \rightarrow \pi\pi \\
 6 & \text{in} & \gamma\gamma \rightarrow \pi\pi \\
 3 & \text{in} & \tau \rightarrow 3\pi\nu_\tau \\
 2 & \text{in} & \pi \rightarrow l\nu\gamma \\
 2 & \text{in} & F_V^\pi(t) \\
 1 & \text{in} & \pi \rightarrow l\nu\gamma^* .
 \end{array} \tag{44}$$

This is quite a manageable number of terms. Still, it remains to be seen whether they allow one to relate different observables in a useful and practical manner. In addition, one may rely on the resonance exchange approximation to estimate some of the relevant constants at this order [17,13], or on sum rules [18].

7.2. Radiative corrections

Once experimental data are sensitive to two-loop contributions, one is forced to also consider radiative corrections due to virtual photons. It has been shown [19] that the electromagnetic corrections to the S-wave scattering lengths are of comparable size to the $O(p^6)$ strong interaction contributions. The relevant effective lagrangian in the strong interaction sector - including real and virtual photons - has been investigated, and several calculations have already been performed [19,20]. In addition, for the not so rare kaon decays being investigated by E865 [32] and by KLOE [34], it will be important to be able to perform these corrections in a systematic manner also for weak interactions. The relevant effective lagrangian has recently been constructed [21].

7.3. Baryon CHPT

Chiral perturbation theory in the baryon sector is not as straightforward as in the meson sector, because the baryon mass does not vanish in the chiral limit and generates thus a new scale. Power counting becomes more difficult - the contribution from loops is not automatically suppressed at low

energy [22]. In the last decade, a special method has been set forward - referred to as heavy baryon chiral perturbation theory (HBCHPT)- to cope with this problem [23]. Recently, it has been shown that one may stay in a manifestly Lorentz invariant framework by simply treating the Feynman integrals in an appropriate manner, such that the infrared singular pieces are singled out automatically, and the polynomial terms that set up the power counting are discarded [24,25]. Using this method, it has been shown [24] that the one-loop expression for pion nucleon scattering, worked out a long time ago [22] in the framework of relativistic chiral perturbation theory, has a low-energy expansions at order p^3 that is identical to the one performed in HBCHPT (in the kinematic region where HBCHPT is applicable [25]).

7.4. Elastic $\pi\pi$ scattering and Roy equations

The interplay between theoretical and experimental aspects of elastic $\pi\pi$ scattering is illustrated in figure 2. As we discussed in subsection 2.1, Weinberg's calculation [5] of the scattering amplitude at leading order in the low-energy expansion gives for the isospin zero S-wave scattering length the value $a_0^0 = 0.16$ in units of the charged pion mass. This differs from the experimentally determined value [6] $a_0^0 = 0.26 \pm 0.05$ by two standard deviations. The one-loop calculation [26] enhances the leading order term to $a_0^0 = 0.20 \pm 0.01$ - the correction goes in the right direction, but the result is still on the low side as far as the present experimental value is concerned. To decide about agreement/disagreement between theory and experiment, one should i) evaluate the scattering lengths in the theoretical framework at order p^6 , and ii) determine them more precisely experimentally. Let me first comment on the *theoretical* work.

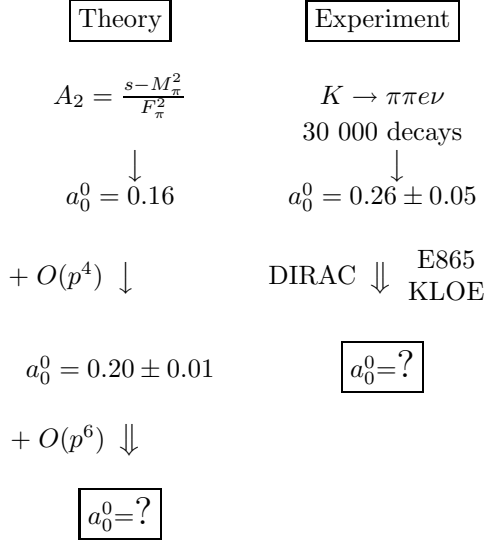


Figure 2. Progress in the determination of the elastic $\pi\pi$ scattering amplitude. References are provided in the text.

The low-energy expansion of the $\pi\pi$ scattering amplitude is of the form

$$A(s, t, u) = A_2 + A_4 + A_6 + O(p^8), \quad (45)$$

where A_n is of order p^n . The tree-level result A_2 is given in (15), and the one-loop expression A_4 may be found in [26]. The two-loop contribution A_6 was worked out in [27]. The amplitude $A_2 + A_4 + A_6$ contains several of the low-energy constants from \mathcal{L}_{eff} :

$$\left. \begin{array}{l} \mathcal{L}_2 : F_\pi, M_\pi \\ \mathcal{L}_4 : \bar{l}_1, \bar{l}_2, \bar{l}_3, \bar{l}_4 \\ \mathcal{L}_6 : \bar{r}_1, \dots, \bar{r}_6 \end{array} \right\} \begin{array}{l} \text{occur in } \pi\pi \rightarrow \pi\pi \\ \text{at order } p^6. \end{array} \quad (46)$$

Once the amplitude is available in algebraic form, it is a trivial matter to evaluate the threshold parameters. To quote an example, the isospin zero S-wave scattering length is of the form

$$\begin{aligned} a_0^0 &= \frac{7M_\pi^2}{32\pi F_\pi^2} \{1 + c_4 x + c_6 x^2\} + O(p^8), \\ x &= \frac{M_\pi^2}{16\pi^2 F_\pi^2}. \end{aligned} \quad (47)$$

The coefficients c_4, c_6 contain the low-energy constants listed in (46). Similar formulae hold for

all other threshold parameters - the explicit expressions for the scattering lengths and effective ranges of the S- and P-waves as well as for the D-wave scattering lengths at order p^6 may be found in [27]. It is clear that, before a numerical value for these parameters can be given, one needs an estimate of the LEC's. The calculation is under way - it is, however, quite involved: One has to solve numerically the Roy-equations [28] with input from the high-energy absorptive part. Second, one assumes that the couplings that describe the mass dependence of the amplitude may be estimated e.g. from resonance exchange. Requiring that the experimental amplitude agrees near threshold with the chiral representation allows one finally to pin down the remaining couplings, as well as the scattering lengths a_0^0 and a_0^2 . The remaining threshold parameters may then be obtained from the Wanders sum rules [29]. The first part of the program is completed, and the report will appear soon [30]. The second part, that will allow us to predict the values of all threshold parameters, is under investigation [31].

On the *experimental* side, several attempts are under way to improve our knowledge of the threshold parameters. The most promising ones among them are i) semileptonic K_{l4} decays with improved statistics, E865 [32] and KLOE [34], and ii) the measurement of the pionium lifetime - DIRAC [35] - that will allow one to directly determine the combination $|a_0^0 - a_0^2|$ of S-wave scattering lengths.

Why are we interested in a precise value of the scattering length a_0^0 ? First, it is one of the few occasions that a quantity in QCD can be predicted rather precisely - which is, of course, by itself worth checking. Second, as has been pointed out in [36], this prediction assumes that the condensate has the standard size in the chiral limit - in particular, it is assumed to be non vanishing. For this reason, the authors of Ref. [36] have reversed the argument and have set up a framework where the condensate is allowed to be small or even vanishing in the chiral limit - the so called generalized chiral perturbation theory. [There is no sign for a small condensate in present lattice calculations [37]. For further investigations of the small condensate scenario see [38,39].] Whereas

the S-wave scattering lengths cannot be predicted in this framework, one may relate their size to the value of the condensate. Hence, measuring a_0^0 , a_0^2 or a combination thereof [35] may allow one to determine the nature of chiral symmetry breaking by experiment [36,40].

7.5. Hadronic atoms

Using the effective lagrangian framework proposed by Caswell and Lepage some time ago [41], the width of ponium in its ground state has been determined [42] at leading and next-to-leading order in isospin breaking and to all orders in the chiral expansion. This result will allow one to evaluate the combination $|a_0^0 - a_0^2|$ with high precision, provided that DIRAC determines the lifetime at the 10% level, as is foreseen [35]. The technique of Caswell and Lepage is very well suited for this purpose, and it is rather easy to carry it over to the case of pion nucleon bound states. Work on this problem is in progress [43]. I refer the reader to Soto's contribution to this conference for an outline of the method.

Acknowledgement

I wish to thank Stephan Narison and his collaborators for the most pleasant and interesting conference and for the friendly atmosphere.

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